OpenFst: An Open-Source, Weighted Finite-State Transducer Library and its Applications to Speech and Language

Part II. *Library Use and Design*

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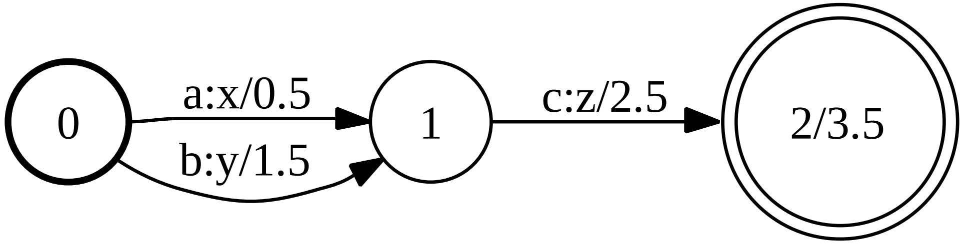
Finite-State Transducer Construction

Input Methods:

- Finite-state transducer:
  - Textual transducer file representation
  - C++ code
  - Graphical user interface
- Regular expressions
- “Context-free” rules
- “Context-dependent” rules

FST Textual File Representation

- **Graphical Representation** `(T.ps)`:



- **Transducer File** `(T.txt)`:

```
0    1    a   x   0.5
0    1    b   y   1.5
1    2    c   z   2.5
2   3.5
```

- **Input Symbols File** (`T.isyms`):

```
a   1
b   2
c   3
```

- **Output Symbols File** (`T.osyms`):

```
x   1
y   2
z   3
```

Compiling, Printing, Reading, and Writing FSTs

- Compiling

```
fstcompile -isymbols=T.isyms -osymbols=T.osyms T.txt T.fst
```

- Printing

```
fstprint -isymbols=T.isyms -osymbols=T.osyms T.fst >T.txt
```

- Drawing

```
fstdraw -isymbols=T.isyms -osymbols=T.osyms T.fst >T.dot
```

- Reading

```
Fst<Arc> *fst = Fst<Arc>::Read(‘‘T.fst’’)
```

- Writing

```
fst.Write(‘‘T.fst’’)
```

C++ FST Construction

```
// A vector FST is a general mutable FST
VectorFst<StdArc> fst;

// Add state 0 to the initially empty FST and make it the start state
fst.AddState(); // 1st state will be state 0 (returned by AddState)
fst.SetStart(0); // arg is state ID

// Add two arcs exiting state 0
// Arc constructor args:  ilabel, olabel, weight, dest state ID
fst.AddArc(0, StdArc(1, 1, 0.5, 1)); // 1st arg is src state ID
fst.AddArc(0, StdArc(2, 2, 1.5, 1));

// Add state 1 and its arc
fst.AddState();
fst.AddArc(1, StdArc(3, 3, 2.5, 2));

// Add state 2 and set its final weight
fst.AddState();
fst.SetFinal(2, 3.5); // 1st arg is state ID, 2nd arg weight
```

OpenFst Design: Arc (transition)

Labels and states may be any integral type; weights may be any class that forms a semiring:

```
struct StdArc {
  typedef int Label;
  typedef TropicalWeight Weight;
  typedef int StateId;

  Label ilabel;                    // Transition input label
  Label olabel;                    // Transition output label
  Weight weight;                   // Transition weight
  StateId nextstate;               // Transition destination state
};
```

OpenFst Design: Tropical Weight

A Weight class holds the set element and provides the semiring operations:

```
class TropicalWeight {
public:
  TropicalWeight(float f) :  value_(f) {}
  static TropicalWeight Zero() { return TropicalWeight(kPositiveInfinity); }
  static TropicalWeight One() { return TropicalWeight(0.0); }
private:
  float value_;
};

TropicalWeight Plus(TropicalWeight x, TropicalWeight y) {
   return w1.value_ < w2.value_ ?  w1 :  w2;
};
```

Similarly, e.g. `LogWeight` and `MinMaxWeight` are defined.

OpenFst Design: Product Weight

This template allows easily creating the product semiring from two (or more) semirings.

```
template <typename W1, typename W2>
class ProductWeight {
public:
  ProductWeight(W1 w1, W2 w2) :  value1_(w1), value2_(w2) {}
  static ProductWeight<W1, W2> Zero() {
    return ProductWeight(W1::Zero(), W2::Zero());
  }
  static ProductWeight<W1, W2> One() {
    return ProductWeight(W1::One(), W2::One());
  }
private:
  float value1_;
  float value2_;
};
```

```
template <typename W1, typename W2>
ProductWeight<W1, W2> Plus(ProductWeight<W1, W2> x, ProductWeight<W1, W2> y) {
  return ProductWeight<W1, W2>(
    Plus(x.value1_, y.value1_),
    Plus(x.value2_, y.value2_));
  };
```

Similarly, e.g. `LexicographicWeight` is defined.

Operation Implementation Types

- **Destructive:** Modifies input; $O(|Q| + |E|)$:

  ```
  StdFst *input = StdFst::Read("input.fst");
  Invert(input);
  ```

- **Constructive:** Writes to output; $O(|Q| + |E|)$:

  ```
  StdFst *input = StdFst::Read("input.fst");
  StdVectorFst output;
  ShortestPath(input, &output);
  ```

- **Lazy (or Delayed):** Creates new Fst; $O(|Q_{visit}| + |E_{visit}|)$:

  ```
  StdFst *input = StdFst::Read("input.fst");
  StdFst *output = new StdInvertFst(input);
  ```

Lazy implementations are useful in applications where the whole machine may not be visited, e.g. Dijsktra (positive weights), pruned search.

Rational Operations

OPERATION	USAGE	DESCRIPTION
Union (Sum)	Union(&A, B); UnionFst<Arc>(A, B); fstunion a.fst b.fst out.fst	contains strings in A and B
Concat (Product)	Concat(&A, B); Concat(A, &B); ConcatFst<Arc>(A, B); fstconcat a.fst b.fst out.fst	contains strings in A followed by B
Closure	Closure(&A, type); ClosureFst<Arc>(A, type); fstclosure in.fst out.fst	$A* = \{\epsilon\} \cup A \cup AA \cup \ldots$

Elementary Unary Operations

OPERATION	USAGE	DESCRIPTION
Reverse	Reverse(A, &B);	reversed strings of A
Invert	Invert(&A); InvertFst<Arc>(A); fstinvert in.fst out.fst	inverse binary relation; swaps input and output labels
Project	Project(&A, type); ProjectFst<Arc>(A, type); fstproject [–project_output] in.fst out.fsa	creates acceptor of just the input or output strings

Fundamental Binary Operations

Operation	Usage	Description
Compose	Compose(A, B, &C); ComposeFst<Arc>(A, B); fstcompose a.fst b.fst out.fst	composition of binary relations
Intersect	Intersect(A, B, &C); IntersectFst<Arc>(A, B); fstintersect a.fsa b.fsa out.fsa	contains strings in both A and B
Difference	Difference(A, B, &C); DifferenceFst<Arc>(A, B); fstdifference a.fsa b.dfa out.fsa	contains strings in A but not in B; B unweighted

Optimization Operations

OPERATION	USAGE	DESCRIPTION
Connect	Connect(&A);	Removes useless states and arcs
RmEpsilon	RmEpsilon(&A); RmEpsilonFst<Arc>(A); fstrmepsilon in.fst out.fst	Equiv. ϵ-free transducer
Determinize	Determinize(A, &B); DeterminizeFst<Arc>(A); fstdeterminize in.fst out.fst	Equiv. deteterministic transducr
Minimize	Mininize(&A); Mininize(&A, &B); fstminimize in.fst out1.fst [out2.fst]	Equiv. minimal det. transducer

Normalization Operations

Operation	Usage	Description
TopSort	TopSort(&A); fsttopsort in.fst out.fst	Topologically sorts an acyclic FST
ArcSort	ArcSort(&A, compare; fstarcsort [–sort-type=$t] in.fst out.fst	Sorts state's arcs given an order relation
Push	Push<Arc, Type>(&A, flags); fstpush [-flags] in.fst out.fst	Creates equiv. pushed/stochastic FST
EpsNormalize	EpsNormalize(A, &B, type); fstepsnormalize [–eps_norm_output] in.fst out.fst	Places path ϵ's after non-ϵ's
Synchronize	Synchronize¡(A, &B); SynchronizeFst<Arc>(A); fstsynchronize in.fst out.fst	Produces monotone epsilon delay

Search Operations

OPERATION	USAGE	DESCRIPTION
ShortestPath	ShortestPath(A, &B, nshortest=1);	N-shortest paths
	fstshortestpath [–nshortest=$n] in.fst out.fst	
ShortestDistance	ShortestDistance(A, &distance);	Shortest distance from initial states
	ShortestDistance(A, &distance, true);	Shortest distance to final states
	fstshortestdistance [–reverse] in.fst [dist.txt]	
Prune	Prune(&A, threshold);	Prunes states and arcs by path weight
	fstprune [–weight=$w] in.fst out.fst	

Traversal Operations

OPERATION	USAGE	DESCRIPTION
Map	Map(&A, mapper); Map(A, &B, mapper); MapFst<IArc, OArc, Mapper>(A, mapper);	Transforms arcs in an FST
Visit	Visit(A, &visitor, &queue);	Visits FST using queue disc.

- Mapper: Class with method:
  - `OArc operator()(const IArc &arc));`
- Visitor: Class with methods e.g.:
  - `bool WhiteArc(StateId s, const Arc &arc));,`
  - `bool GreyArc(StateId s, const Arc &arc));,`
  - `bool BlackArc(StateId s, const Arc &arc));`
- Queue: Class with methods e.g.:
  - `bool Enqueue(StateId s);`
  - `StateId Head();`
  - `void Dequeue();`
  - `bool Empty();`

Example: FST Application - Shell-Level

```
# The FSTs must be sorted along the dimensions they will be joined.
# In fact, only one needs to be so sorted.
# This could have instead been done for "model.fst" when it was
created.
$ fstarcsort --sort_type=olabel input.fst input_sorted.fst
$ fstarcsort --sort_type=ilabel model.fst model_sorted.fst

# Creates the composed FST
$ fstcompose input_sorted.fst model_sorted.fst comp.fst

# Just keeps the output label
$ fstproject --project_output comp.fst result.fst

# Do it all in a single command line
$ fstarcsort --sort_type=ilabel model.fst |
fstcompose input.fst - | fstproject --project_output result.fst
```

Example: FST Application - C++

```
// Reads in an input FST.
StdFst *input = StdFst::Read("input.fst");

// Reads in the transduction model.
StdFst *model = StdFst::Read("model.fst");

// The FSTs must be sorted along the dimensions they will be joined.
// In fact, only one needs to be so sorted.
// This could have instead been done for "model.fst" when it was created.
ArcSort(input, StdOLabelCompare());
ArcSort(model, StdILabelCompare());

// Container for composition result.
StdVectorFst result;

// Create the composed FST
Compose(*input, *model, &result);

// Just keeps the output labels
Project(&result, PROJECT_OUTPUT);
```

Example: Shortest-Distance with Various Semirings

- Tropical Semiring:

```
Fst<StdArc> *input = Fst<StdArc>::Read("input.fst");
vector<StdArc::Weight> distance;
ShortestDistance(*input, &distance);
```

- Log Semiring:

```
Fst<LogArc> *input = Fst::Read("input.fst");
vector<LogArc::Weight> distance;
ShortestDistance(*input, &distance);
```

- Right String Semiring:

```
typedef StringArc<TropicalWeight, STRING_RIGHT> SA;
Fst<SA> *input = Fst::Read("input.fst");
vector<SA::Weight> distance;
ShortestDistance(*input, &distance);
```

- Left String Semiring:

```
ERROR: ShortestDistance:  Weights need to be right distributive
```

Example: Expectation Semiring

Let $\mathbb{K}$ denote $(\mathbb{R} \cup \{+\infty, -\infty\}) \times (\mathbb{R} \cup \{+\infty, -\infty\})$. For pairs (x_1, y_1) and (x_2, y_2) in $\mathbb{K}$, define the following :

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2, y_1 + y_2)\\ (x_1, y_1) \otimes (x_2, y_2) &= (x_1 x_2, x_1 y_2 + x_2 y_1)\end{aligned}$$

The system $(\mathbb{K}, \oplus, \otimes, (0,0), (1,0))$ defines a commutative semiring.

This semiring combined with the composition and shortest-distance algorithms has be used e.g. to compute the relative entropy between probabilistic automata [C. Cortes, M. Mohri, A. Rastogi, and M. Riley. On the Computation of the Relative Entropy of Probabilistic Automata. *International Journal of Foundations of Computer Science*, 2007.]:

$$D(A\|B) = \sum_x [\![A]\!](x) \log [\![A]\!](x) - \sum_x [\![A]\!](x) \log [\![B]\!](x).$$

This algorithm is trivially implemented in the OpenFst Library.

OpenFst Design: Fst (generic)

```
template <class Arc>
class Fst {
public:
  virtual StateId Start() const = 0;                // Initial state
  virtual Weight Final(StateId) const = 0;          // State's final weight
  static Fst<Arc> *Read(const string filename);
}
```

OpenFst Design: State Iterator

```
template <class F>
class StateIterator {
public:
  explicit StateIterator(const F &fst);
  virtual ~StateIterator();
  virtual bool Done();                    // States exhausted?
  virtual StateId Value() const;          // Current state Id
  virtual void Next();                    // Advance a state
  virtual void Reset();                   // Start over
}
```

OpenFst Design: Arc Iterator

```
template <class F>
class ArcIterator {
public:
  explicit ArcIterator(const F &fst, StateId s);
  virtual ~ArcIterator();
  virtual bool Done();                                  // Arcs exhausted?
  virtual const Arc &Value() const;                     // Current arc
  virtual void Next();                                  // Advance an arc
  virtual void Reset();                                 // Start over
  virtual void Seek(size_t a);                          // Random access
}
```

OpenFst Design: MutableFst

```
template <class Arc>
class MutableFst : public Fst<Arc> {
public:
  void SetStart(StateId s);                  // Set initial state
  void SetFinal(StateId s, Weight w);        // Set final weight
  void AddState();                           // Add a state
  void AddArc(StateId s, const Arc &arc);    // Add an arc
}
```

OpenFst Design: Mutable Arc Iterator

```
template <class F>
class MutableArcIterator {
public:
  explicit MutableArcIterator(F *fst, StateId s);
  virtual ~MutableArcIterator();
  virtual bool Done();                              // Arcs exhausted?
  virtual const Arc &Value() const;                 // Current arc
  virtual void Next();                              // Advance an arc
  virtual void Reset();                             // Start over
  virtual void Seek(size_t a);                      // Random access
  virtual void SetValue(const Arc &arc);            // Set current arc
}
```

OpenFst Design: Invert (Destructive)

```
template <class Arc> void Invert(MutableFst<Arc> *fst) {
  for (StateIterator< MutableFst<Arc> > siter(*fst);
    !siter.Done();
    siter.Next()) {
      StateId s = siter.Value();
      for (MutableArcIterator< MutableFst<Arc> > aiter(fst, s);
        !aiter.Done();
        aiter.Next()) {
          Arc arc = aiter.Value();
          Label l = arc.ilabel;
          arc.ilabel = arc.olabel;
          arc.olabel = l;
          aiter.SetValue(arc);
      }
  }
}
```

Easier to use `Map` for this case.

OpenFst Design: Invert (Lazy)

```
template <class Arc> class InvertFst :  public Fst<Arc> {
public:
  virtual StateId Start() const { return fst_->Start(); }
  ...
private:
  const Fst<Arc> *fst_;
}

template <class F> Arc ArcIterator<F>::Value() const {
  Arc arc = arcs_[i_];
  Label l = arc.ilabel;
  arc.ilabel = arc.olabel;
  arc.olabel = l;
  return arc;
}
```

Easier to use `MapFst` for this case.

Transition Representation

- We have represented a transition as:

$$e \in Q \times (\Sigma \cup \{\epsilon\}) \times (\Delta \cup \{\epsilon\}) \times \mathbb{K} \times Q.$$

  - Treats input and output symmetrically
  - Space-efficient single output-label per transition
  - Natural representation for composition algorithm

- Alternative representation of a transition:

$$e \in Q \times (\Sigma \cup \{\epsilon\}) \times \Delta^* \times \mathbb{K} \times Q.$$

  or equivalently,

$$e \in Q \times (\Sigma \cup \{\epsilon\}) \times \mathbb{K}' \times Q, \qquad \mathbb{K}' = \Delta^* \times \mathbb{K}.$$

  - Treats string and $\mathbb{K}$ outputs uniformly
  - Natural representation for weighted transducer determinization, minimization, label pushing, and epsilon normalization.

Operations Using Alternative Transition Representation

- We can use the alternative transition representation with:

```
typedef ProductWeight<StringWeight, TropicalWeight> GallicWeight;
```

- Weighted transducer determinization becomes:

```
Fst<StdArc> *input = Fst::Read("input.fst");
// Converts into alternative transition representation
MapFst<StdArc, GallicArc> gallic(*input, ToGallicMapper);
WeightedDeterminizeFst<GallicArc> det(gallic);
// Ensures only one output label per transition (functional input)
FactorWeightFst<GallicArc> factor(det);
// Converts back from alternative transition representation
MapFst<GallicArc> result(factor, FromGallicMapper);
```

- Efficiency is not sacrificed given the lazy computation and an efficient string semiring representation.

- Weighted transducer minimization, label pushing and epsilon normalization are similarly implemented easily using the generic (acceptor) weighted minimization, weight pushing, and epsilon removal algorithms.

Operation Options

- Example Options:

```
typedef RhoMatcher< SortedMatcher<StdFst> > RM;

ComposeFstOptions<StdArc, RM> opts;

opts.matcher1 = new RM(fst1, MATCH_NONE, kNoLabel);
opts.matcher2 = new RM(fst2, MATCH_INPUT, kNoLabel);

StdComposeFst cfst(fst1, fst2, opts);
```

- Many operations optionally take similar option arguments.

Composition: Matcher Design

- Matchers can find and iterate through requested labels at FST states; principal use in composition matching.
- Matcher Form:

```
template <class F>
class Matcher {
typedef typename F::Arc Arc;

public:
  void SetState(StateId s);     // Specifies current state
  bool Find(Label label);       // Checks state for match to label
  bool Done() const;            // No more matches
  const Arc& Value() const;     // Current arc
  void Next();                  // Advance to next arc
};
```

Matchers

- Predefined Matchers:

NAME	DESCRIPTION
SortedMatcher	Binary search on sorted input
RhoMatcher<M>	ρ symbol handling; templated on underlying matcher
SigmaMatcher<M>	σ symbol handling; templated on underlying matcher
PhiMatcher<M>	φ symbol handling; templated on underlying matcher

- The *special symbols* referenced above behave as:

	CONSUMES NO SYMBOL	CONSUMES SYMBOL
MATCHES ALL	ϵ	σ
MATCHES REST	φ	ρ